

Philosophy 220A  
Symbolic Logic I

Instructor: Richard Johns

## Answers to Problem Set 6

Total: 55 marks

1. The negation seems to be “If G then  $\neg Q$ ”, i.e. “If the NHL increases the size of the goal then Luongo will keep playing in the league anyway”. [1 mark]

2.  $(A \rightarrow B) \vee (\neg A \rightarrow C)$

- (i) Either if Alice will win then Betty will be pleased, or if Alice doesn't win then Chris will be pleased. [1 mark]

(ii)

A	B	C	$(A \rightarrow B) \vee (\neg A \rightarrow C)$			
T	T	T	T	T	F	T
T	T	F	T	T	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	F

↑  
(1)

[4 marks]

[4 marks]

They should do *either* the proof or the table, so only give marks for one of them.

1.	
2. $A \vee \neg A$	✓ ▽ Taut Con:
3. ▽ $A$	
4. ▽ $\neg A$	
5. $\perp$	✓ ▽ $\perp$ Intro: 3,4
6. $C$	✓ ▽ $\perp$ Elim: 5
7. $\neg A \rightarrow C$	✓ ▽ $\rightarrow$ Intro: 4-6
8. $(A \rightarrow B) \vee (\neg A \rightarrow C)$	✓ ▽ $\vee$ Intro: 7
9. ▽ $\neg A$	
10. ▽ $A$	
11. $\perp$	✓ ▽ $\perp$ Intro: 9,10
12. $B$	✓ ▽ $\perp$ Elim: 11
13. $(A \rightarrow B)$	✓ ▽ $\rightarrow$ Intro: 10-12
14. $(A \rightarrow B) \vee (\neg A \rightarrow C)$	✓ ▽ $\vee$ Intro: 13
15. $(A \rightarrow B) \vee (\neg A \rightarrow C)$	✓ ▽ $\vee$ Elim: 3-8,9-14,2

(iii) Does your English sentence in (ii) look like a logical truth? I'm not sure. Not really, but it's such a weird sentence I'm not even sure what it means. [1 mark]

7.2 They are TT equivalent [3 marks]

A	B	$\neg(A \rightarrow B)$		$A \wedge \neg B$
T	T	F	T	F F
T	F	T	F	T T
F	T	F	T	F F
F	F	F	T	F T
		(1)	(2)	

N.B. Unlike in Qu. 1, it seems here that to deny a conditional requires one to believe its antecedent.

7.5 Not TT equivalent. [3 marks]

A	B	C	$(A \wedge B) \rightarrow C$		$A \rightarrow (B \vee C)$	
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	F
			(1)		(2)	

7.12 These should be labelled 2 to 20, of course. My apologies. [1 mark each]

T 1.  $(\text{LeftOf}(a, d) \vee \text{RightOf}(a, d)) \rightarrow \text{Cube}(a)$

T 2.  $\text{Small}(c) \rightarrow \text{RightOf}(c, a)$

T 3.  $\text{Tet}(e) \rightarrow (\text{RightOf}(e, b) \leftrightarrow \text{FrontOf}(e, b))$

T 4.  $\text{BackOf}(c, a) \wedge \text{FrontOf}(c, e)$

T 5.  $\text{Cube}(a) \vee \text{Cube}(c) \vee \text{Cube}(e)$

T 6.  $\text{Larger}(b, a) \wedge \text{Larger}(b, e)$

T 7.  $\text{SameShape}(d, b) \rightarrow \text{SameSize}(d, b)$

T 8.  $\neg \text{Tet}(c) \rightarrow \text{Cube}(b)$

T 9.  $(\text{Tet}(a) \vee \text{Tet}(c)) \rightarrow (\text{Cube}(b) \vee \text{Cube}(d))$

T 10.  $\text{Large}(a) \leftrightarrow \text{Large}(e)$

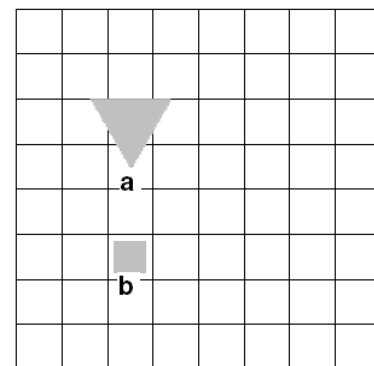
**8.20** [3 marks]

- |                                 |                                   |
|---------------------------------|-----------------------------------|
| 1. $B \rightarrow C$            |                                   |
| 2. $\nabla A \wedge B$          |                                   |
| 3. $B$                          | ✓ $\nabla \wedge$ Elim: 2         |
| 4. $C$                          | ✓ $\nabla \rightarrow$ Elim: 3,1  |
| 5. $(A \wedge B) \rightarrow C$ | ✓ $\nabla \rightarrow$ Intro: 2-4 |

**8.21** [4 marks]

T 1.  $\text{Adjoins}(a,b) \rightarrow \neg \text{Large}(a)$

F 2.  $(\text{Adjoins}(a, b) \vee \text{SameCol}(a, b)) \rightarrow \neg \text{Large}(a)$



## 8.29 [10 marks]

You might deduct one mark if they write down  $P \vee \neg P$ , citing Theorem 6.33, since this wasn't specifically allowed.

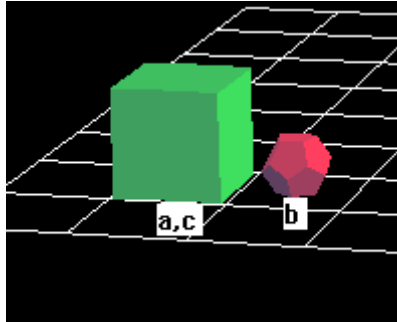
1.	
2. $\nabla P \rightarrow Q$	
3. $\nabla \neg(\neg P \vee Q)$	
4. $\nabla \neg P$	
5. $\neg P \vee Q$	✓ $\nabla \vee$ Intro: 4
6. $\perp$	✓ $\nabla \perp$ Intro: 5,3
7. $P$	✓ $\nabla \neg$ Intro: 4-6
8. $Q$	✓ $\nabla \rightarrow$ Elim: 2,7
9. $\neg P \vee Q$	✓ $\nabla \vee$ Intro: 8
10. $\perp$	✓ $\nabla \perp$ Intro: 9,3
11. $\neg P \vee Q$	✓ $\nabla \neg$ Intro: 3-10
12. $\nabla \neg P \vee Q$	
13. $\nabla P$	
14. $\nabla \neg P$	
15. $\perp$	✓ $\nabla \perp$ Intro: 13,14
16. $Q$	✓ $\nabla \perp$ Elim: 15
17. $\nabla Q$	
18. $Q$	✓ $\nabla$ Reit: 17
19. $Q$	✓ $\nabla \vee$ Elim: 12,14-16,17-
20. $P \rightarrow Q$	✓ $\nabla \rightarrow$ Intro: 13-19
21. $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$	✓ $\nabla \leftrightarrow$ Intro: 2-11,12-20

### 8.33 [10 marks]

1. $(\text{Horned}(c) \rightarrow (\text{Elusive}(c) \wedge \text{Magical}(c))) \wedge (\neg \text{Horned}(c) \rightarrow (\neg \text{Elusive}(c) \wedge \neg \text{Magical}(c)))$	
2. $\neg \text{Horned}(c) \rightarrow \neg \text{Mythical}(c)$	
3. $\text{Horned}(c) \rightarrow (\text{Elusive}(c) \wedge \text{Magical}(c))$	✓ ▾ $\wedge$ Elim: 1
4. $\neg \text{Horned}(c) \rightarrow (\neg \text{Elusive}(c) \wedge \neg \text{Magical}(c))$	✓ ▾ $\wedge$ Elim: 1
5. ▾ $\text{Horned}(c)$	
6. $\text{Elusive}(c) \wedge \text{Magical}(c)$	✓ ▾ $\rightarrow$ Elim: 3,5
7. $\text{Magical}(c)$	✓ ▾ $\wedge$ Elim: 6
8. $\text{Magical}(c) \vee \text{Mythical}(c)$	✓ ▾ $\vee$ Intro: 7
9. ▾ $\text{Magical}(c) \vee \text{Mythical}(c)$	
10. ▾ $\neg \text{Horned}(c)$	
11. $\neg \text{Elusive}(c) \wedge \neg \text{Magical}(c)$	✓ ▾ $\rightarrow$ Elim: 10,4
12. $\neg \text{Magical}(c)$	✓ ▾ $\wedge$ Elim: 11
13. $\neg \text{Mythical}(c)$	✓ ▾ $\rightarrow$ Elim: 2,10
14. ▾ $\text{Magical}(c)$	
15. $\perp$	✓ ▾ $\perp$ Intro: 14,12
16. ▾ $\text{Mythical}(c)$	
17. $\perp$	✓ ▾ $\perp$ Intro: 16,13
18. $\perp$	✓ ▾ $\vee$ Elim: 9,14-15,16-17
19. $\neg \neg \text{Horned}(c)$	✓ ▾ $\neg$ Intro: 10-18
20. $\text{Horned}(c)$	✓ ▾ $\neg$ Elim: 19
21. $\text{Horned}(c) \leftrightarrow (\text{Magical}(c) \vee \text{Mythical}(c))$	✓ ▾ $\leftrightarrow$ Intro: 5-8,9-20

## 8.44

The argument is invalid since, in the world shown, the premises are true and the conclusion false. [5 marks]



T 1.  $\text{Adjoins}(a, b) \wedge \text{Adjoins}(b, c)$

T 2.  $\text{SameRow}(a, c)$

F 3.  $a \neq c$